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REPORT

# Medium frequency directional aerials

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Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION August 1973

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# MEDIUM FREQUENCY DIRECTIONAL AERIALS

L.K. Bradley, B.A., M.I.E.E.

## Summary

*The principal design considerations for highly directional m.f. aerial arrays are presented together with a detailed calculation relating to a particular example of an in-line four-element directional array. Techniques for providing phase control and power splitting are also considered.*

Issued under the authority of

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Research Department, Engineering Division,
BRITISH BROADCASTING CORPORATION

(RA-111)

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## 1. Introduction

The advent of local broadcasting on medium frequencies (m.f.) coupled with the scarcity of available channels has led to proposals for the use of directional aerial systems to enable a number of stations to use the same frequency without causing unacceptable mutual interference.

In the past the BBC has employed directional patterns for some m.f. stations by positioning one or more parasitic radiators to modify the pattern of a single driven aerial but the degree of radiation pattern control by this means is limited. Much better control can be achieved by driving a number of radiators, modifying the amplitudes and phases of the aerial currents to obtain the required pattern. A study was made to investigate the problems involved and the most important considerations and conclusions are the subject of this report. As an example, a four-element in-line aerial arrangement is given which was designed to satisfy the requirement of a service templet (Manchester Local Radio).

## 2. The case for directional aerials

Assuming service areas spaced 300 km or more apart, local services could be provided without serious mutual interference during the day, but at night power would have to be reduced to combat sky-wave interference, in some cases seriously restricting the night-time service area. In the United Kingdom, with smaller separation of transmitters, the problem is proportionately greater unless directional arrays are employed. Apart from controlling the horizontal radiation pattern it is also possible to control the vertical radiation pattern to some extent thereby reducing sky-wave interference.

Directional aerial techniques were first discussed in the 1930's by G.H. Brown[1] and others and many broadcasting stations in the United States[2,3] and elsewhere[4] have used m.f. directional arrays for many years. In the United States in particular directional m.f. arrays have been in use for twenty to thirty years.[2] Some stations maintain the same power and pattern for day and night operation, while others use different day and night arrays and transmitter powers.[2,3]

## 3. Radiation pattern synthesis

A number of approaches to the problem of designing an array to fit a required templet have been proposed[5,6,7,8] and these are discussed in some detail by Laporte.[5] Most of these methods require a certain amount of 'cut and try' and one method,[7] based on a Fourier analysis, while always providing a theoretical solution, does not necessarily provide a practical or economical solution, particularly for asymmetric patterns.

If it is considered necessary or desirable to employ only two or three radiators Smith and Hutton[9] give two tower patterns having 45° steps in phasing and spacing up to an array length of four wavelengths and 15° steps in phasing and spacing up to an array length of one wavelength, while Smith[10] gives 568 two-tower patterns with spacings up to four wavelengths and 14,592 three-tower patterns with the three towers on radials spaced 45° the towers being displaced up to one wavelength from the centre of the radials. As an example the templet for the Manchester local radio station (Fig. 1) is almost symmetrical and originally a rectangular arrangement was considered but it appeared difficult to obtain the broad service area required and at the same time achieve a sharp cut off near ±90° to the line of the array. The method adopted was to multiply the pattern of the two-aerial arrangement giving a broad service area and a poor back-to-front ratio by a pattern giving a good back-to-front ratio — selecting suitable patterns from those given in the N.A.B. Engineering Handbook[11] (see Fig. 2).

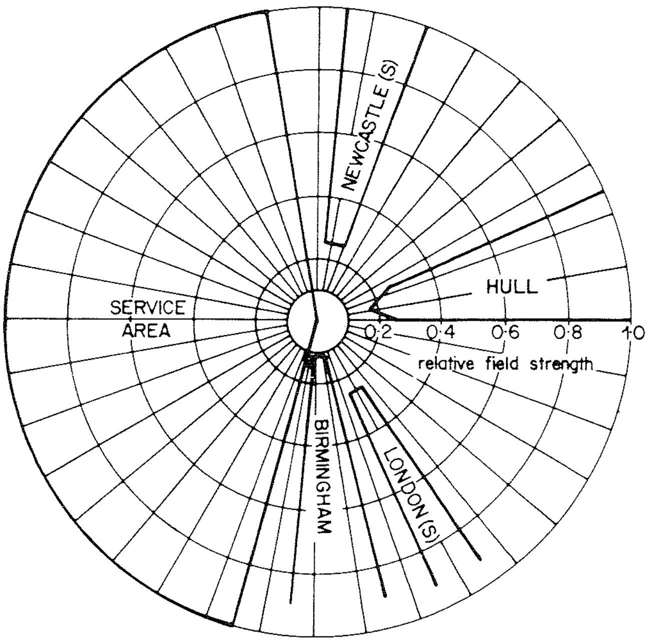


*Fig. 1 - Specified Manchester templet*

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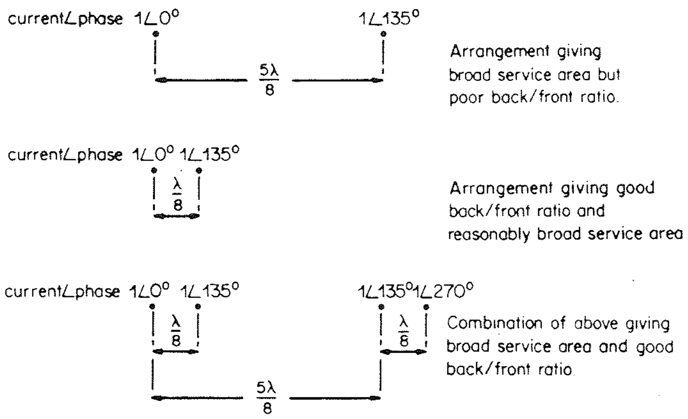



*Fig. 2 - Development of required templet*

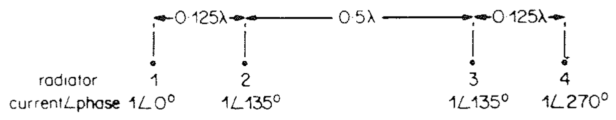



*Fig. 3 - Proposed aerial system for Manchester*

It may be possible to introduce inductance by coiling the tower lighting or probe connector leads to tune out these strays.

## 4. Mast impedance

Typical mast radiators are of triangular steel lattice construction, three $1^{11}/32''$ O.D. members spaced 1'3" centre to centre. In practice $\lambda/4$ radiators would generally be employed.

The equivalent cylindrical mast would have a diameter of approximately 1·05 ft[12] which, for a height of 160 ft ($\lambda/4$ at 1457 kHz), gives a height/radius ratio of 304.

The self and mutual impedances were taken from the King-Middleton[13] tables for a height/radius ratio of 259 and the operational base impedances calculated assuming equal currents in all radiators with the spacing and phasing shown in Fig. 3.

The base impedances will be modified by base insulator capacitance and stray capacitances due to tower lighting, lightning protector circuits, monitoring probes etc.

## 5. Aerial matching units

The operational base impedances of the radiators will be matched to 50Ω coaxial cables by means of L-type transformers (Fig. 4) which will be housed close to the mast base. The base impedance is made real by paralleling it with a suitable reactance and this effective resistance is then matched to 50Ω (Fig. 4(*a*)).

## 6. Power splitting

The power splitting network proposed in this report is shown in Fig. 5. L-type transformers (Fig. 4(*c*)) transform the 50Ω loads presented by the radiators and their corresponding matching units to values of resistance $R_1$, $R_2$, $R_3$, $R_4$ which ensures the required power distribution. The capacitances ($C_r$) are combined in one capacitance $C$.

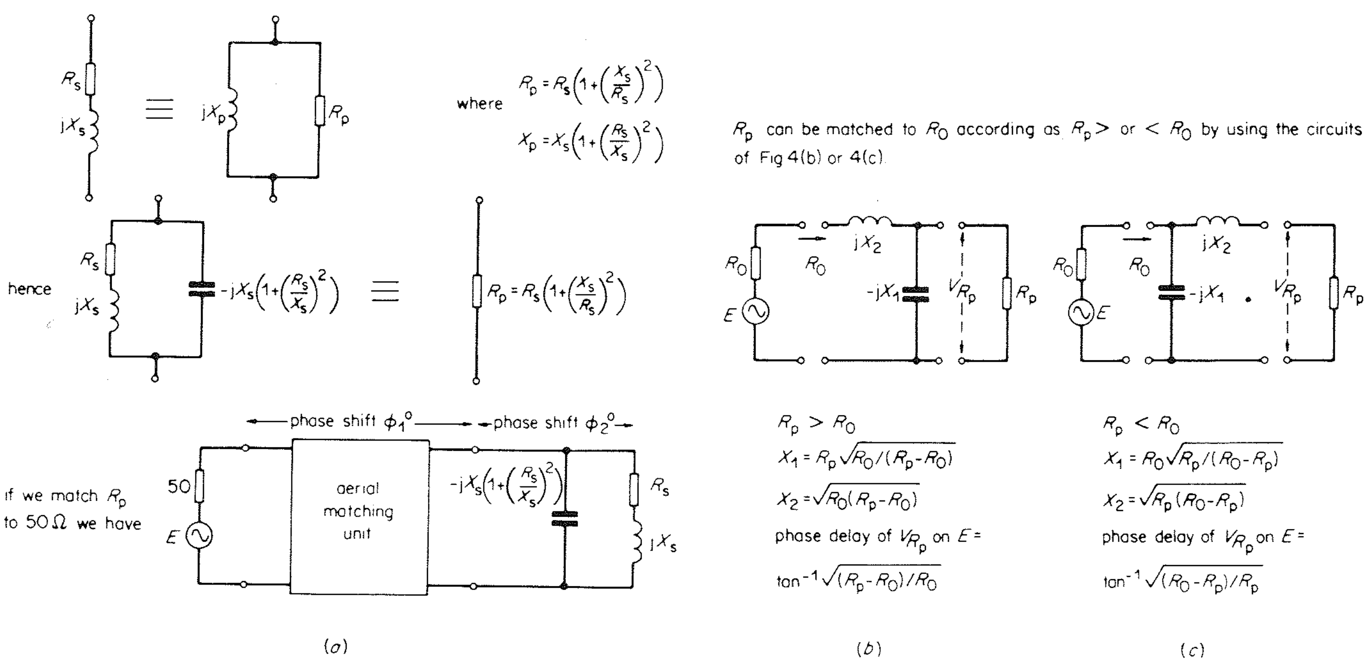



*Fig. 4 - L-type transformers*

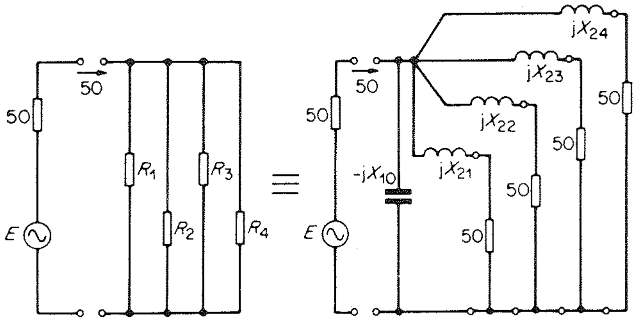



*Fig. 5 - Power dividing network*

## 7. Phasing

Phase shifts occur in the power splitting and aerial matching networks and to achieve the required phase relationships between the aerial currents additional phase shifts are required. Some of these could be obtained by using unequal feeder lengths but it would still be necessary to make final adjustments with phase shift networks (Fig. 6) to trim out the differences between the calculated and actual phase shifts obtained in the various networks. In the case of the proposed Manchester aerial equal length feeders have been assumed but, if necessary, unequal lengths could be used, allowance being made for the different attenuations of the cables by alterations in the power dividing network.

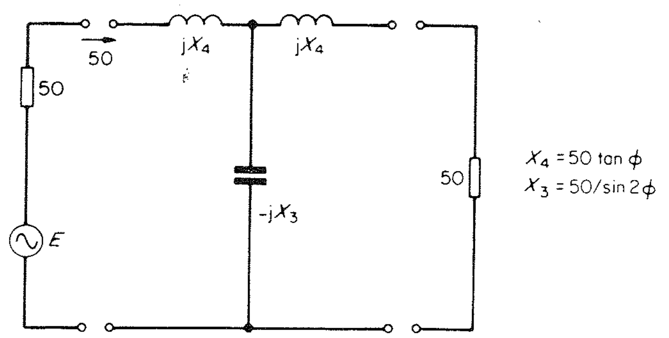
## 8. Stability of amplitude and phase

With the arrangement of Fig. 3, radiation patterns were calculated assuming:

(a) $\pm 10^\circ$ phase errors on aerials 2, 3 and 4

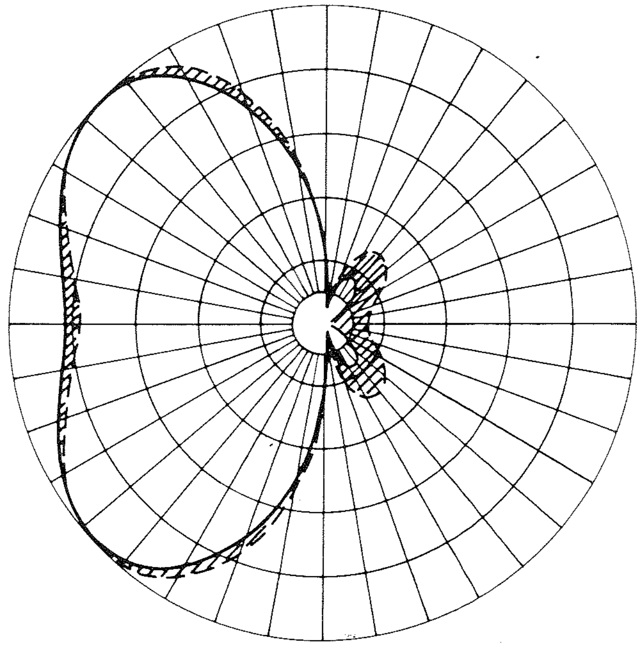
(b) $\pm 5^\circ$ phase errors on aerials 2, 3 and 4

From these results it was found that satisfactory performance requires not more than $\pm 5^\circ$ phase error. Further patterns were then calculated with $\pm 5^\circ$ phase error and ±0·5 dB amplitude error and from this selection it appears that $\pm 5^\circ$ phase error and ±0·5 dB could be tolerated (Fig. 7).




With this network inserted as shown the power in the load is $E^2/4 \times 50$ and the phase delay is $2\phi$.

*Fig. 6 - Phase shift network*



Dotted lines indicate effect of variations of ±5° and ±0·5dB in relative amplitudes and phases of currents in radiators.

*Fig. 7 - Calculated h.r.p. of proposed Manchester aerial*

## 9. Feeders

Using radiators approximately a quarter of a wavelength high will result in low input impedances and feeders of the order of 50Ω will be suitable. Since it is very important with directional arrays to prevent unwanted radiation, coaxial feeders with a solid outer are required.

## 10. Installation of directional m.f. aerial systems

It is necessary to take great care in the installation of directional m.f. systems to avoid instability of operation.[3,11,14,15] All towers should be as nearly identical as possible with respect to height, orientation, guy wires, subdivision, monitoring loops etc. All sections of towers should be well bonded before painting. No flexible conduits should be used for any purpose near the aerial circuits. Conduits used for lights etc., should be bonded to the earth system at regular intervals. If the transmission lines are laid in troughs the outer conductors should be bonded to the earth system every 20 ft; preferably to the heavy strap between the towers. If the towers are metal-fenced these fences must be in position and grounded before impedance measurements are made. Only silver soldering, brazing or welding should be used for all joints.

It is particularly important that changes in ground conductivity should not affect the tower impedance and some writers[3,16] recommend a raised earth mat at the base of the radiators. Also great care must be taken to avoid corrosion in ground wires and ground interconnections.

## 11. Monitoring

If the minimum field requirement is not more than

20 dB below the maximum it is sufficient to monitor the amplitudes and phases of the currents in the radiators. This is preferably done by placing loops on the tower so that the base current in each radiator is directly monitored.[11] The loops are connected to the monitoring circuits in the transmitter house by equal lengths of coaxial cable having continuous outer sheaths. For low-power operation these loops can be insulated from the mast but the connecting cables must be earthed at regular intervals between the mast and transmitter building. Amplitudes and phases of currents must be checked regularly to ensure maintenance of the required pattern.

## 12. Setting up arrays

Since the radiators are all coupled by their mutual impedances, changes in any of the radiator currents will affect the impedances of all elements. Hence the power setting, phase shifting and matching adjustments are all interdependent. Brown[17] avoids this by first measuring the self and mutual impedances by conventional bridge methods, calculating the input impedances with the required driving currents, replacing the radiators with dummy impedances and adjusting all the circuits at low power, and finally replacing the dummy loads by their respective aerials 'If normal care has been exercised, the system should be in close adjustment'.

More recently Colligan[18] and Surutka[19] have described methods of measuring the operating impedances of and adjusting medium-wave directive arrays at low power, in which the power division and phase shifting networks are isolated from the radiators by buffer amplifiers.

## 13. The aerial arrangement for the example

As mentioned in Section 9 quarter-wave radiators are to be preferred.

At 1457 kHz $\lambda$ = 205·9 m; therefore a quarter-wave radiator would have an $h/a$ ratio of 320. Williams[20] indicates that such a mast would have a velocity ratio $\approx$ 0·95 thereby reducing the quarter-wave height to 160·4 ft and the $h/a$ ratio to 304. Using the formula

$$Z_o = 60(\log_e \frac{2h}{a} - 1)$$

this ratio gives an impedance of 325Ω.

The nearest $h/a$ value in the King-Middleton tables[13] is 259, corresponding to an impedance of 318Ω, and in the absence of measured values the King-Middleton figures were used for the self and mutual impedances of the radiators.

Fig. 8 shows the result of measurements made on a B.I.C.C. mast of the type mentioned above compared with

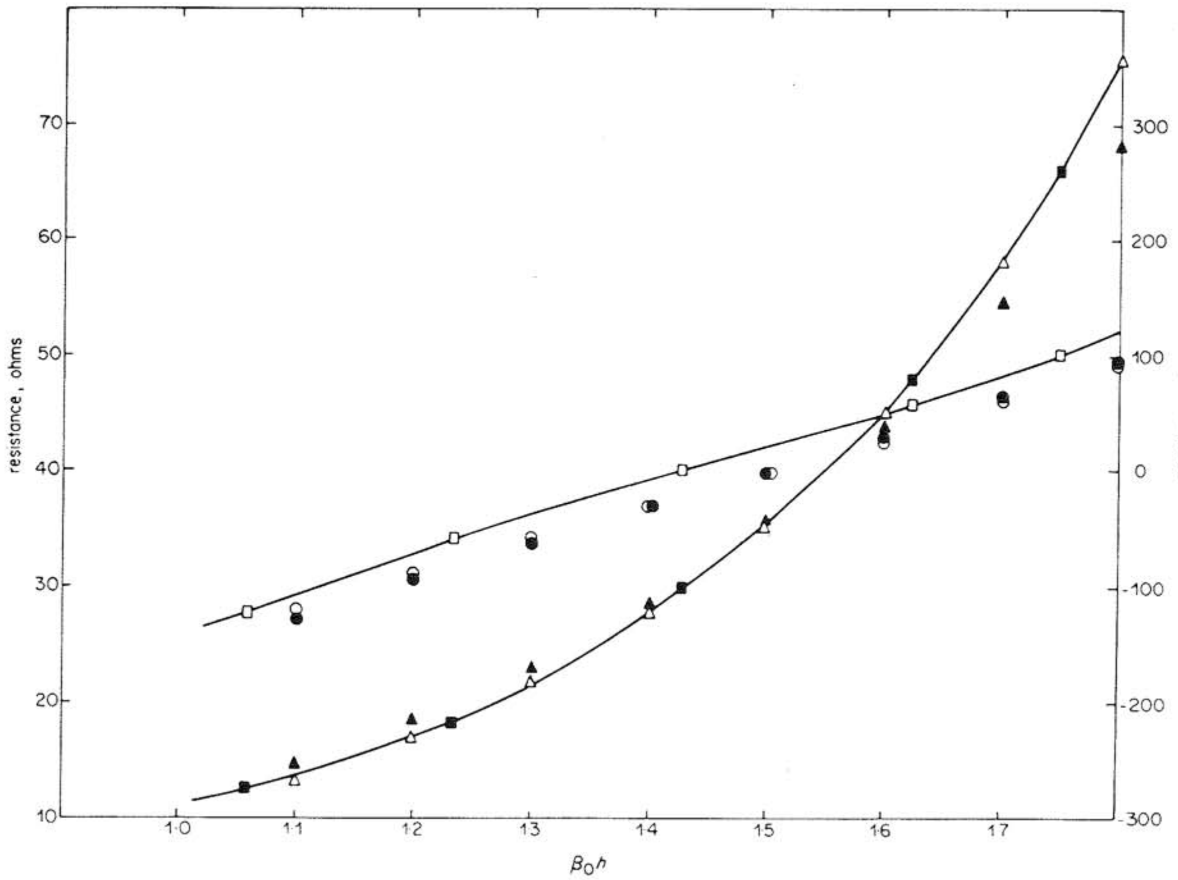



*Fig. 8 - Measured impedance of typical mast radiator*

▲ Series resistance ) mast with h/a = 259
● Series reactance ) taken from King-Middleton tables
△ Series resistance ) of same mast with 50 pF
○ Series reactance ) across feed point
■ Series resistance ) Measured values
□ Series reactance ) B.I.C.C. mast

the impedances assumed in the calculation. It also shows the effect of a small amount of capacitance across the base of the mast. The curves for measured resistance and reactance have been shifted slightly to the right – in other words the effective height of the mast was increased by approximately 4%.

The required templet is shown in Fig. 1 which assumes a maximum e.r.p. of 16 kW. The computed horizontal radiation pattern (h.r.p.) based on the arrangement of Fig. 3 is shown in Fig. 7. The shaded areas show the effect of ±5° errors in relative current phases together with ±0·5 dB errors in relative amplitudes. To achieve optimum protection for Birmingham the line of the proposed array would be in the direction 278° – 98° ETN.

Interference with Newcastle or London would be via the sky-wave and at 1457 kHz and distances up to 400 km E and F-layer reflections need be considered. At distances corresponding to the service areas of London and Newcastle such reflections would arise from signals transmitted at angles of elevation varying between 25° and 70°. Calculations of sky-wave radiation using a Research computer programme indicate that provided nominal radiator current phases and amplitudes are maintained protection should be satisfactory (Fig. 9).

The Appendix contains calculations of radiator input impedances, matching networks, power dividing networks and phase shifters – equal length feeders being assumed to all radiators. This is probably the preferred arrangement for a directional system since any change on one feeder would most probably be matched by corresponding changes in the other feeders tending to preserve the current amplitude and phase relationships. Fig. 10 shows the proposed arrangement in schematic form. All elements are shown separately although in practice series inductors would in some cases be combined and paralleled components replaced by a single component.

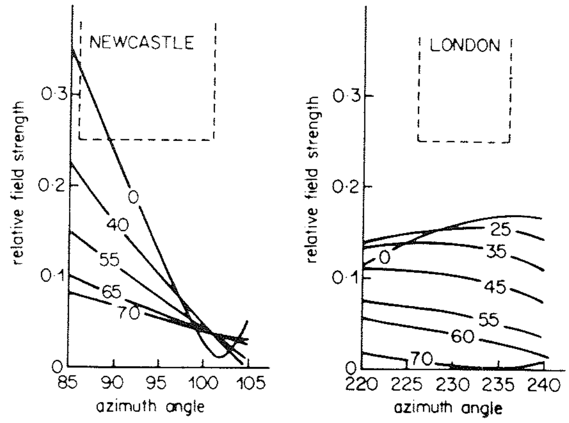



*Fig. 9 - Sky-wave propagation*

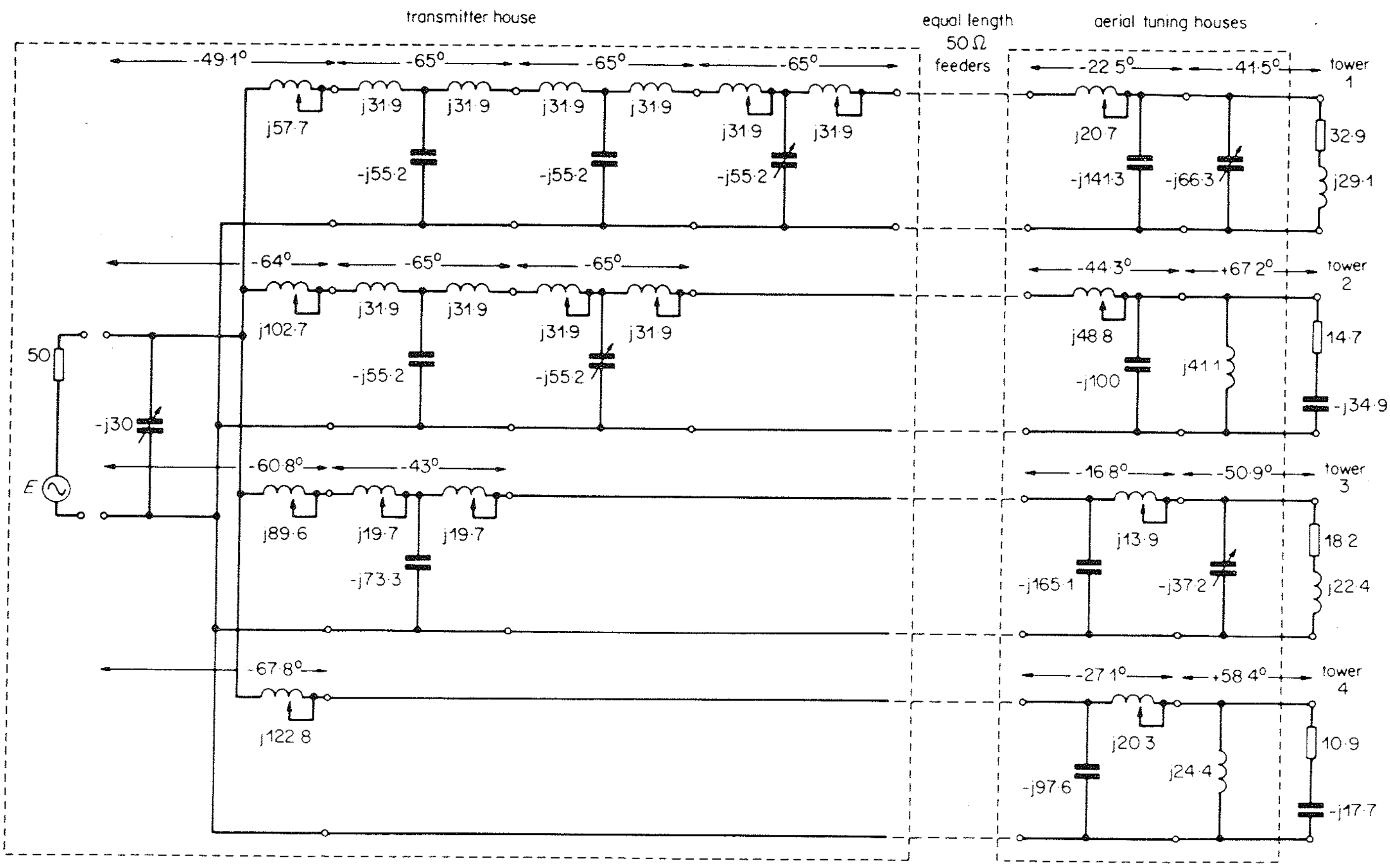



*Fig. 10 - Radiator amplitude and phasing arrangements*

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## Appendix

### 1. Calculation of radiator input impedances

We assume an array as in Fig. 3 using quarter wavelength radiators and neglect base insulator and other capacitances (assuming that they can be tuned out).

At 1457 kHz

$$\lambda = 205{\cdot}9 \text{ m} = 675{\cdot}5 \text{ ft}$$

The radiators will have a velocity ratio of the order of 0·95. Hence for a $\lambda/4$ radiator

$$h = 48{\cdot}9 \text{ m} = 160{\cdot}4 \text{ ft}$$

An equivalent cylindrical mast has a diameter of 1·05 ft (see 3) giving

$$\frac{2h}{d} = 305{\cdot}5$$

also

$$\beta_0 h = \frac{2\pi h}{\lambda} = 1{\cdot}49$$

From the King-Middleton tables[13] (p.174) by interpolation

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = 35 - \text{j}1{\cdot}0$$

If $b$ is the spacing between two radiators then for

$$b = \frac{\lambda}{8},\ \frac{\lambda}{2},\ \frac{5\lambda}{8},\ \frac{3\lambda}{4}$$

$$\beta_0 b = 0{\cdot}785,\ 3{\cdot}14,\ 3{\cdot}93,\ 4{\cdot}71$$

From the King-Middleton tables[13] (p.300) by interpolation

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = 29{\cdot}5 - \text{j}6{\cdot}0 \quad \because \beta b = 0{\cdot}785$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = -11 - \text{j}3{\cdot}5 \quad \because \beta b = 3{\cdot}93$$

$$Z_{14} = Z_{41} = -10{\cdot}3 + \text{j}4{\cdot}3 \quad \because \beta b = 4{\cdot}71$$

$$Z_{23} = Z_{32} = -5{\cdot}5 - \text{j}12{\cdot}0 \quad \because \beta b = 3{\cdot}14$$

The currents in the radiators are

$$I_1 = I(1)$$

$$I_2 = I(-0{\cdot}707 + \text{j}0{\cdot}707)$$

$$I_3 = I(-0{\cdot}707 + \text{j}0{\cdot}707)$$

$$I_4 = I(-\text{j})$$

The voltage $E_r$ across the r$^{\text{th}}$ radiator is given by

$$E_r = \sum_{n=1}^{4} Z_{rn} \cdot I_n$$

Hence

$$Z_r = \frac{E_r}{I_r} = \sum_{n=1}^{4} Z_{rn} \frac{I_n}{I_r}$$

$$\therefore Z_1 = \frac{E_1}{I_1} = Z_{11} + Z_{12}\frac{I_2}{I_1} + Z_{13}\frac{I_3}{I_1} + Z_{14}\frac{I_4}{I_1}$$

$$= 35 - \text{j}1{\cdot}0 + (29{\cdot}5 - \text{j}6{\cdot}0)(-0{\cdot}707 + \text{j}0{\cdot}707)$$

$$+ (-11 - \text{j}3{\cdot}5)(-0{\cdot}707 + \text{j}0{\cdot}707)$$

$$+ (-10{\cdot}3 + \text{j}4{\cdot}3)(-\text{j})$$

$$= 32{\cdot}9 + \text{j}29{\cdot}1$$

Similarly

$$Z_2 = (29{\cdot}5 - \text{j}6{\cdot}0)\left(\frac{1}{-0{\cdot}707 + \text{j}0{\cdot}707}\right) + 35 - \text{j}1{\cdot}0$$

$$+ (-5{\cdot}5 - \text{j}12{\cdot}0) + (-11 - \text{j}3{\cdot}5)\left(\frac{-\text{j}}{-0{\cdot}707 + \text{j}0{\cdot}707}\right)$$

$$= 14{\cdot}7 - \text{j}34{\cdot}9$$

$$Z_3 = (-11 - \text{j}3{\cdot}5)\left(\frac{1}{-0{\cdot}707 + \text{j}0{\cdot}707}\right) + (-5{\cdot}5 - \text{j}12{\cdot}0)$$

$$+ (35 - \text{j}1{\cdot}0) + (29{\cdot}5 - \text{j}6{\cdot}0)\left(\frac{-\text{j}}{-0{\cdot}707 + \text{j}0{\cdot}707}\right)$$

$$= 18{\cdot}2 + \text{j}22{\cdot}4$$

$$Z_4 = (-10{\cdot}3 + \text{j}4{\cdot}3)\text{j} + (-11 - \text{j}3{\cdot}5)\left(\frac{-0{\cdot}707 + \text{j}0{\cdot}707}{-\text{j}}\right)$$

$$+ (29{\cdot}5 - \text{j}6{\cdot}0)\left(\frac{-0{\cdot}707 + \text{j}0{\cdot}707}{-\text{j}}\right) + 35 - \text{j}1{\cdot}0$$

$$= 10{\cdot}9 - \text{j}17{\cdot}7$$

If we transform the series representations of $Z_1$, $Z_2$, $Z_3$ and $Z_4$ to their parallel equivalents and tune out the reactive components we are left with the resistive parts $R_p$ to be matched to 50Ω with L-type transformers in the

*TABLE I*

| | Series components (Ω) | Parallel components (Ω) | Type* | $X_2$ * (Ω) | $X_1$ * (Ω) | $\phi_1$ * (°) | $\phi_2$ * (°) | Total phase shift (°) |
|---|---|---|---|---|---|---|---|---|
| $Z_1$ | 32·9 + j29·1 | 58·6/+ j66·3 | (a) | +20·7 | +141·3 | −22·5 | −41·5 | −64 |
| $Z_2$ | 14·7 − j34·9 | 97·6/− j41·1 | (a) | +48·8 | +100 | −44·3 | +67·2 | +22·9 |
| $Z_3$ | 18·2 + j22·4 | 45·8/+ j37·2 | (b) | +13·9 | +165·1 | −16·8 | −50·9 | −67·7 |
| $Z_4$ | 10·9 − j17·7 | 39·6/− j24·4 | (b) | +20·3 | +97·6 | −27·1 | +58·4 | +31·3 |

* see Fig. 4.

Aerial Tuning Houses (Fig. 4). These circuits will introduce phase shifts and the total phase shift between the driving voltage and the aerial currents will be the sum of phase shifts in the Aerial Matching Units and in the aerials themselves. Table I shows the calculated component values for the aerial matching units and total resultant phase shifts.

With equal current in all four radiators the required power distribution ratios will be

32·9 : 14·7 : 18·2 : 10·9

Hence if we transform the 50Ω loads, presented by the radiators with their matching units, into $R_1$, $R_2$, $R_3$ and $R_4$

$$\text{where } \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \frac{1}{R_4} : : 32{\cdot}9 : 14{\cdot}7 : 18{\cdot}2 : 10{\cdot}9 \qquad (1)$$

$$\text{and} \qquad \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{50} \qquad (2)$$

we shall have the required power distribution because the power in any load $R_r$ will be given by

$$P_r = \frac{E^2}{4R_r}$$

$$\text{From (1)} \qquad \frac{1}{R_2} = \frac{14{\cdot}7}{32{\cdot}9} \cdot \frac{1}{R_1} \text{ etc.}$$

$$\therefore \text{ from (2)} \qquad \frac{1}{50} = \frac{1}{R_1}\left(1 + \frac{14{\cdot}7}{32{\cdot}9} + \frac{18{\cdot}2}{32{\cdot}9} + \frac{10{\cdot}9}{32{\cdot}9}\right)$$

whence $R_1$ = 116·6Ω, $R_2$ = 260·9Ω, $R_3$ = 210·7Ω and

$R_4$ = 351·8Ω

Since we need to transform 50Ω to higher values, L sections of the type 4(c) will be required – $R_p$ being replaced by 50Ω and the input resistance required $R_r$ instead of 50Ω giving the following:

*TABLE II*

| Required load (Ω) | | $X_1$ (Ω) | $X_2$ (Ω) | Phase shift (°) |
|---|---|---|---|---|
| $R_1$ | 116·6 | 101 | 57·7 | −49·1 |
| $R_2$ | 260·9 | 127 | 102·7 | −64·0 |
| $R_3$ | 210·7 | 117·5 | 89·6 | −60·8 |
| $R_4$ | 351·8 | 143·2 | 122·8 | −67·8 |

The complete network is shown in Fig. 5 – $X_{2r}$ being the value of $X_2$ corresponding to $R_r$ and the four values of $X_1$ being combined in one reactance $-jX_{10}$ where

$$\frac{1}{X_{10}} = \sum \frac{1}{X_{1r}}$$

$$\therefore \; X_{10} = 30$$

**Phase shifters**

From Tables I and II the total phase shifts in power divider, matching units and aerial are as follows:

| aerial feed | Total phase shift (°) |
|---|---|
| 1 | −113·1 |
| 2 | −41·1 |
| 3 | −128·5 |
| 4 | −36·5 |

giving phase shifts relative to 1 of

0, +72, −15·4 and +76·6 degrees

while the required relative phases are (c.f. Fig. 3)

0, +135, +135, +270

If we insert a delay of 193·4 degrees in the feed to 1 we have:

0, +265·4, +178, +270

Then insert delays of 130·4 degrees in 2 and 43 degrees in 3 and we have the required phase distribution.

If the network of Fig. 6 is matched to 50Ω the power in the load will be $E^2/4 \times 50$ and the phase shift $-2 \tan^{-1} X_4/50$.

Therefore denoting the phase shift by $-2\phi$ we have

$$X_4 = 50 \tan\phi \text{ and it can be shown that}$$

$$X_3 = 50/\sin 2\phi$$

The phase delay required in the feed to 1 is 193·4° ≈ 3 x 65° and that in 2 130·4° ≈ 2 x 65°. Hence since the networks will require adjustment these can all be to the same design. We shall have

$$X_4 = 31{\cdot}9\Omega$$

$$X_3 = 55{\cdot}2\Omega$$

For the 43° phase shift network

$$X_4 = 19{\cdot}7\Omega$$

$$X_3 = 73{\cdot}3\Omega$$

Alternatively the 193·4° phase shift can be obtained with two 90° sections and ≈ 14° and 130·4 with 90° and ≈ 40.

For 90° sections

$$X_4 = 50\Omega$$

$$X_3 = 50\Omega$$

For 14° sections

$$X_4 = 6{\cdot}1\Omega$$

$$X_3 = 206{\cdot}7\Omega$$

and for a 40° section

$$X_4 = 18{\cdot}2\Omega$$

$$X_3 = 77{\cdot}8\Omega$$

SMW/MS